

JP.103. Let $x, y, z > 0$ be positive real numbers. Then in triangle ABC with semiperimeter s and inradius r .

$$\frac{x}{y+z} \cot^2 \frac{A}{2} + \frac{y}{z+x} \cot^2 \frac{B}{2} + \frac{z}{x+y} \cot^2 \frac{C}{2} \geq 18 - \frac{s^2}{2r^2}$$

Proposed by D.M. Băţineţu – Giurgiu – Romania, Martin Lukarevski – Skopje

JP.104. Let r_a, r_b, r_c be the exradii, h_a, h_b, h_c the altitudes and m_a, m_b, m_c the medians of a triangle ABC with semiperimeter s , circumradius R and inradius r . Then

$$\frac{r_a^2}{h_b m_c} + \frac{r_b^2}{h_c m_a} + \frac{r_c^2}{h_a m_b} \geq \frac{54r^2}{s^2 - r^2 - 4Rr}$$

Proposed by D.M. Băţineţu – Giurgiu – Romania, Martin Lukarevski – Skopje

JP.105. Let $m > 0$ and F be the area of the triangle ABC . Then

$$\frac{a^{m+2}}{b^m + c^m} + \frac{b^{m+2}}{c^m + a^m} + \frac{c^{m+2}}{a^m + b^m} \geq 2\sqrt{3}F.$$

Proposed by D.M. Băţineţu – Giurgiu – Romania, Martin Lukarevski – Skopje

PROBLEMS FOR SENIORS

SP.091. Prove that for all positive real numbers a, b, c, d

$$\begin{aligned} \frac{a^2}{a+b+c} + \frac{b^2}{b+c+d} + \frac{c^2}{c+d+a} + \frac{d^2}{d+a+b} &\geq \\ &\geq \frac{a+b+c+d}{3} + \frac{4(2a+b-2c-d)^2}{27(a+b+c+d)} \end{aligned}$$

Proposed by Nguyen Viet Hung – Hanoi – Vietnam

SP.092. Prove that for all positive real numbers a, b, c

$$\begin{aligned} \text{a. } \frac{a^2}{a+b} + \frac{b^2}{b+c} + \frac{c^2}{c+a} &\geq \frac{a+b+c}{2} + \frac{(b-c)^2}{2(a+b+c)} \\ \text{b. } \frac{a^2}{b+c} + \frac{b^2}{c+a} + \frac{c^2}{a+b} &\geq \frac{a+b+c}{2} + \frac{(a+b-2c)^2}{2(a+b+c)} \end{aligned}$$

Proposed by Nguyen Viet Hung – Hanoi – Vietnam

SP.093. Prove that in any triangle ABC the following inequality holds

$$\frac{(b+c)a}{m_a^2} + \frac{(c+a)b}{m_b^2} + \frac{(a+b)c}{m_c^2} \geq 8$$

Proposed by Nguyen Viet Hung – Hanoi – Vietnam